

Critical wetting of a class of nonequilibrium interfaces: A computer simulation study

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Critical wetting transitions under nonequilibrium conditions are studied numerically and analytically by means of an interface-displacement model defined by a Kardar-Parisi-Zhang equation, plus some extra terms representing a limiting, short-ranged attractive wall. Its critical behavior is characterized in detail by providing a set of exponents for both the average height and the surface order-parameter in one dimension. The emerging picture is qualitatively and quantitatively different from recently reported mean-field predictions for the same problem. Evidence is shown that the presence of the attractive wall induces an anomalous scaling of the interface local slopes.

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I. INTRODUCTION

Much scientific effort has gone into the study of equilibrium wetting since, in the late 1970s, Cahn introduced the idea that it can be described as a phase transition [1]. Among the various theoretical approaches developed, interface displacement models have proved particularly useful [2]. Within this perspective, the focus is on the interface that separates two coexisting (bulk) phases confined by a wall or substrate, and the wetting transition corresponds to the unbinding of the interface from the wall. This happens upon a rise of the temperature when the wall adsorbs preferentially one of the phases leading to a divergence of the thickness of the adsorbed layer. The dynamics of such an interface can be described at a coarse-grained level by the following continuum stochastic growth equation [3]:

$$\frac{\partial h(\mathbf{x}, t)}{\partial t} = D\nabla^2 h - \frac{\delta V(h)}{\delta h} + \eta(\mathbf{x}, t). \quad (1)$$

Here, $h(\mathbf{x}, t)$ is the local height of the interface from the wall, the regions $y > h(\mathbf{x})$ and $y < h(\mathbf{x})$ corresponding to the two bulk phases. D is the interfacial tension coefficient and η is a Gaussian white noise with zero mean and variance $\langle \eta(\mathbf{x}, t) \eta(\mathbf{x}', t') \rangle = 2\sigma \delta(\mathbf{x} - \mathbf{x}') \delta(t - t')$ that mimics thermal fluctuations. $V(h)$ accounts for the net interaction between the wall and the interface and its form depends on the nature of the forces between the particles in the bulk phases and with the wall, its rigorous derivation from microscopic Hamiltonians being far from trivial. If all the interactions are short ranged, one may take in the limit of large h at phase coexistence [2]

$$V(h) = \int dx \left(b(T) e^{-h(x)} + \frac{c}{2} e^{-2h(x)} \right), \quad (2)$$

where T is the temperature [4]. The amplitude $c > 0$ is a repulsion whereas $b(T)$ vanishes linearly with the mean-field wetting temperature, T_w , as $T - T_w$, and can represent either an effective repulsion or attraction between the interface and the wall (see Fig. 1). At sufficiently low temperatures, $b < 0$, the equilibrium thickness of the wetting layer as given

by the stationary configurational average $\langle h \rangle$ is finite (pinned interface). This corresponds to an attractive potential (see Fig. 1). As the temperature is raised the potential becomes less attractive and eventually, above a certain value $b = b_w$, it no longer binds the interface and $\langle h \rangle$ diverges. Within mean-field approximation, ignoring spatial correlations, $\langle h \rangle$ follows from $\partial V(h) / \partial h = 0$, whereby one finds for an attractive wall ($b < 0$) $\langle h \rangle = \ln(-c/b)$ and, consequently, a critical wetting transition takes place as $b \rightarrow b_w = 0$. Recently, effective short ranged, equilibrium critical wetting showing mean-field-like exponents seems to have been experimentally observed [5].

Extensions of equilibrium, interface displacement models to nonequilibrium conditions have only been recently addressed and constitute a topic of ongoing research activity [6,7]. Supplementing Eq. (1) with the most relevant nonequilibrium nonlinear term $\lambda(\nabla h)^2$ [8], leads to a natural generalization of Eq. (1) that assumes that the velocity of the interface depends on its local slope,

$$\frac{\partial h(x, t)}{\partial t} = D\nabla^2 h + \lambda(\nabla h)^2 + b e^{-h} + c e^{-2h} + \eta(x, t), \quad (3)$$

which is a Kardar-Parisi-Zhang (KPZ) interface [8] interacting via a short-ranged potential with a wall. KPZ interfaces have a nonzero average velocity, $v = \lambda \langle (\nabla h)^2 \rangle$, and hence steady-state interfaces move on average thereby favoring one

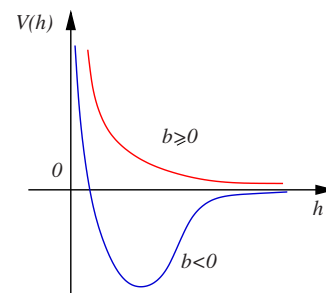


FIG. 1. (Color online) Mean-field binding potential for positive and negative values of b . For $b < 0$ the potential is attractive, exhibiting a well near the wall.

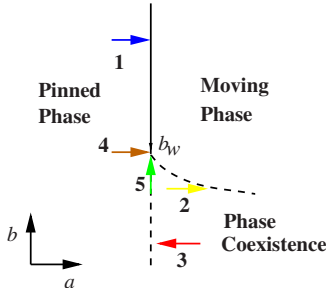


FIG. 2. (Color online) Schematic phase diagram for $\lambda < 0$ and a lower wall in the a - b plane [Eq. (4)]. The vertical line corresponds to the critical value $a = a_c$, and the arrows denote the different types of transitions explained in the text. Path 1: Complete wetting (upon approaching a_c). Path 2: Nontrivial depinning transition at $a^*(b)$. Path 3: First-order pinning transition. Path 4: Multicritical complete wetting. Path 5: Critical wetting.

of the phases over the other. Wetting, on the other hand, by definition occurs at coexistence, i.e., zero average velocity of the free (no wall) interface. This agrees with the thermodynamic picture that at bulk coexistence any arbitrary fraction of the system may be in one phase, with the remainder in the other. Therefore, a constant $a_c = -v$ needs to be included in Eq. (3) to study wetting transitions driven by the wall. The nonequilibrium analog of equilibrium critical wetting corresponds to the depinning transition at a_c as $b \rightarrow b_w^-$. Clearly, for $\lambda = 0$ and $a_c = 0$, the model reduces to the equilibrium one. In equilibrium, a constant force term, a , in the interfacial equation measures the deviation from bulk coexistence (i.e., it represents the chemical potential difference between the two phases) while here it plays a similar role by balancing the force exerted by the KPZ nonlinearity on the tilted parts of the interface, thereby guaranteeing that the average velocity of the free interface is zero.

Owing to the lack of $h \leftrightarrow -h$ symmetry of the KPZ dynamics, it is necessary to specify either the relative position of the wall with respect to the interface, upper or lower, for a fixed sign of λ , or reversely, the sign of λ after a wall position has been arbitrarily chosen. In earlier studies of nonequilibrium (complete) wetting (see below), these two different physical situations lead to the existence of by-now-well-documented two different universality classes, called multiplicative noise 1 (MN1) and multiplicative noise 2 (MN2), respectively [9]. Here, we take $\lambda < 0$ in Eq. (3) (i.e., the critical wetting counterpart of MN1) which has shown to have a much richer phenomenology than that of $\lambda > 0$ in other studies of nonequilibrium wetting [9,10]. The analysis of the case $\lambda > 0$ will be addressed elsewhere.

Thus defined, this model system, arguably the simplest nonequilibrium one, has served for the study of universality issues in nonequilibrium wetting. In particular, by fixing $b > b_w$, i.e., in the presence of a repulsive wall, and letting $a \rightarrow a_c^-$ (path 1 in Fig. 2) nonequilibrium complete wetting transitions (MN1) were investigated. For $b < b_w$ (attractive wall) and varying a there is a rich phenomenology: the pinned and the depinned phases lose their stability at different values of a , giving rise to a continuous depinning transition at $a^*(b) > a$ in the directed-percolation universality class

(see Fig. 2, path 2) [11], and a first-order phase transition along path 3, at $a = a_c$. In the broad interval $a_c < a < a^*(b)$ both phases coexist (see [9] and references therein). Tricritical behavior along path 4 was analyzed by Ginelli *et al.* [12], and a preliminary study of nonequilibrium critical wetting (path 5) was presented in [13].

In this paper we investigate numerically and analytically nonequilibrium critical wetting (path 5 in Fig. 2) as defined by Eq. (3). The focus will be on one-dimensional systems only. Higher system dimensionalities were studied in [10] by a mean-field analytic approximation to Eq. (3) which revealed the existence of three different regimes of scaling behavior. Their connection with our findings (or the lack of it) is discussed in the last section.

II. MODELS AND OBSERVABLES

Our continuous model is defined by the stochastic growth equation

$$\frac{\partial h(x,t)}{\partial t} = D\nabla^2 h + \lambda(\nabla h)^2 + a_c + be^{-h} + ce^{-2h} + \eta(x,t), \quad (4)$$

with, as explained above, $a_c = -\lambda\langle(\nabla h)^2\rangle$. At the critical wetting transition, i.e., as b approaches b_w from below, the average stationary thickness of the wetting layer diverges continuously as $\langle h \rangle \sim |b - b_w|^{\beta_h}$, where β_h is a critical exponent. At $b = b_w$, $\langle h(t) \rangle \sim t^{\theta_h}$ for asymptotically long times. Two other exponents we study are the dynamic and the correlation length exponents, z and ν , respectively, defined through their usual expressions $\xi(t) \sim t^{1/z}$, $\xi \sim |b - b_w|^{-\nu}$, where ξ is the correlation length. They are related to the previous ones by the scaling form $\theta_h = \beta_h / z\nu$.

Some of these exponents may be written in terms of known KPZ exponents. In particular, since at $b = b_w$ the interface is asymptotically free, the dynamic exponent retains its one-dimensional free KPZ value $z = 3/2$ [7].

Also, as we illustrate now, θ_h is given by the exponent characterizing the growth of the interfacial width in the KPZ, $W(t) \sim t^{\theta_w}$, and therefore, $\theta_h = \theta_w = 1/3$ in one dimension. This can be understood as follows: interfacial fluctuations are cutoff owing to the presence of the wall, as a result of which there is an effective fluctuation-induced repulsion between the wall and the interface. The latter can be estimated by noting that the wall makes itself felt when the mean interfacial separation $\langle h \rangle$ is of the same order as the average extent of the interface fluctuations, δh . From $\delta h \sim \xi^\zeta$, where ζ is the usual KPZ roughness exponent, and the definition of ν we find that the effective repulsion force has the form $h^{-1/\zeta\nu}$ and is therefore long ranged in $d=1$ where $\zeta = 1/2 > 0$. Comparing this force with the deterministic one in the Langevin equation (4), which is short ranged, it is straightforward to conclude that in $d=1$ fluctuations dominate the unbinding of the interface. As fluctuations are governed, in the regime where the interface is asymptotically free, by the growth exponent of the KPZ, then $\theta_h = 1/3$. This result has been verified in our computer simulations (see below). Note that this, as well as any other exponent computed exactly at

the critical point, is path independent and, therefore, holds also for complete wetting.

Of more interest is the behavior of the surface order parameter defined as $\langle e^{-h} \rangle$ or, equivalently as the density of local contacts between the interface and the wall. Indeed, considering Eq. (4) with $D=-\lambda$, the change of variables $h = -\ln n$ leads to

$$\frac{\partial n(x,t)}{\partial t} = D\nabla^2 n - a_c n - bn^2 - cn^3 + n\eta(x,t), \quad (5)$$

which is a multiplicative noise Langevin equation for the surface order parameter [9] to be interpreted in the Stratonovich sense [14]. For sufficiently low values of $b < b_w$ the interface remains pinned and the stationary density of locally pinned sites at the wall is high ($n \leq 1$). As the transition is approached (increasing b following path 5 in Fig. 2), the stationary density of pinned segments goes to zero in a continuous manner as $\langle n(b, t=\infty) \rangle \sim |b - b_c|^{\beta_n}$. At $b = b_w$, the interface depins and therefore $\langle h(t) \rangle$ diverges and $\langle n(t) \rangle$ vanishes with the characteristic exponent $\langle n(b = b_w, t) \rangle \sim t^{-\theta_n}$. Obviously, as said before, θ_n is common to paths 4 and 5 in Fig. 2, and the value $\theta_n = 0.5(1)$ has been reported previously [12]. Here we focus on the determination of the path-dependent exponents β_n , β_n , and ν . Before proceeding further, we refer the reader to [15] for a detailed analysis of the critical behavior of the above observables in equilibrium critical wetting.

To study numerically the model defined above, owing to well-documented numerical instability problems [16], it is more convenient to integrate numerically Eq. (5) than the equivalent form Eq. (4). Equation (5) can be efficiently integrated by means of a recently introduced split-step scheme specifically designed to deal with Langevin equations with nonadditive noise [17]. Setting $D = -\lambda = 0.1$, $\sigma = 1$ we can use the result $a_c = 0.143\ 668(3)$ obtained from previous investigations of the analogous nonequilibrium complete-wetting transition [18]. As will be illustrated below, estimates of the critical exponents are severely hindered by uncertainties in the value of a_c .

To circumvent this problem and to confirm universality we have carried out simulations of a discrete interfacial growth model which (i) in the absence of walls is known to belong to the KPZ universality class, (ii) has been successfully used in nonequilibrium complete wetting analyses [19] and, most importantly, (iii) allows for an exact determination of the velocity of the free interface, and therefore permits to extract the critical exponents with good accuracy. To be more specific, we consider a single-step-plus-wall model (SSW) defined as follows. At time t interface positions above sites i of a one-dimensional line of length L are given by integer height variables $h_t(i)$, satisfying the solid-on-solid constraint $|h_t(i) - h_t(i+1)| = 1$. Initially, we take $h_0(2i) = 0$ and $h_0(2i+1) = 1$. New height configurations are generated by choosing at random a site i and growing it to $h_t(i) \rightarrow h_t(i) + 2$ if and only if a local minimum existed at i . It can be shown that this rule generates a KPZ-like interface with $\lambda = -1/2$ moving with an asymptotic long time, average velocity $v = (1+L)/(2L)$ [20]. Additionally, taking advantage of the exact knowl-

edge of v , the interface is globally pulled down by one unit every L/v growth trials, in such a way that the interface has zero average velocity. A wall at $h=0$ is then introduced by precluding the interface from overtaking the wall that is behind it. This is achieved by implementing the previous global, downwards movement as $h_t(i) = |h_t(i) - 1|$ for each i . Finally, in order to implement an attractive wall, the growing rates at the bottom layer are reduced from 1 to $1-q$ with $0 \leq q \leq 1$. The parameter q represents the short-ranged attraction exerted by the wall on the interface. If $q=1$ no growth is possible at the local minima located at the wall and hence an interface at the wall does not move, whereas if $q=0$ the short-range attraction is switched off.

In all, the SSW algorithm is as follows: (i) A site i is randomly chosen and grown from $h(i)$ to $h(i)+2$ if a local minimum exists at i [$h(i+1) + h(i-1) - 2h(i) = 2$]. This is done with probability 1 if $h(i) > 0$, or with probability $1-q$ if $h(i) = 0$, with $0 \leq q \leq 1$. No action is taken if i does not correspond to a minimum. Time is increased by $1/v$ after L of such attempts. (ii) Every L/v growth trials $h(i) = |h(i) - 1|$ for each i . Given that L/v is generally not an integer, this is done by using $[L/v]$ with probability $L/v - [L/v]$ and $[L/v] + 1$ with probability $[L/v] + 1 - L/v$, where $[\cdot]$ denotes the integer part. Periodic boundary conditions are imposed. By tuning the parameter q a critical wetting transition is observed.

III. NUMERICAL RESULTS

We next summarize our main findings for both the discrete model (SSW) and the stochastic differential equation (SDE) (5).

To determine the critical points, q_w (for the SSW) and b_w (for the SDE), we take a system size as large as possible ($L = 2^{17}$ here), plot the order parameter $\langle n(t) \rangle$ versus t in a double logarithmic scale, and look for the separatrix between curves converging to a constant value and those bending downward (not shown). In this way, the critical values $q_w = 0.4445(5)$ (SSW) and $b_w = -1.0815(9)$ (SDE) are determined. In Fig. 3 results are shown in $\ln\text{-}\ln$ for the time decay of the order parameter $\langle n(t) \rangle$ at the critical point of the SSW model for different system sizes ranging from $L = 2^7$ to $L = 2^{17}$. From the slope of a straight-line fit to the lowest curve one finds $\theta_n^{\text{SSW}} = 0.49(2)$ [$\theta_n^{\text{SDE}} = 0.50(5)$ for the SDE; not shown], where the error is computed by comparing the slopes corresponding to the upper and lower bounds of q_w (b_w). The inset of Fig. 3 shows estimates of the crossover times, $t_\times(L)$, from time decay to saturation as a function of the system size. Using $t_\times(L) \sim L^z$ we find the temporal exponent values $z^{\text{SSW}} = 1.4(1)$, $z^{\text{SDE}} = 1.3(2)$ (not shown), which are in agreement with those previously reported in [12] following path 4 of Fig. 2, and compatible with the theoretical considerations described above.

From the scaling of the saturation value at the critical point, $\langle n_{\text{sat}}(q = q_w) \rangle$ for different system sizes one can determine (see Fig. 4) $\beta_n/\nu = 0.74(1)$ [$\beta_n/\nu = 0.6(1)$ for the SDE]. A direct estimation of β_n is also possible by measuring the order-parameter stationary value for the largest available system size upon approaching the critical point from below.

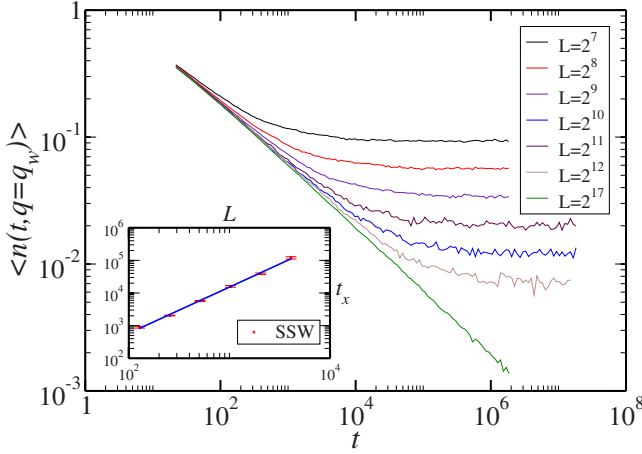


FIG. 3. (Color online) Main: Time decay of the surface order parameter at $q=q_w$ for simulations of the SSW model at system sizes (from top to bottom) $L=2^7, 2^8, 2^9, 2^{10}, 2^{11}, 2^{12}$, and 2^{17} from the lowest curve $\theta_h^{\text{SSW}}=0.49(2)$. Inset: The crossover times to saturation as a function of the system size lead to $z^{\text{SSW}}=1.4(1)$. The error bars indicate the estimated uncertainties in the crossing point of fits of the initial decay and the saturating behavior to straight lines, on the ln-ln plot.

This is shown in Fig. 5 again for both the SSW and the SDE. We find $\beta_n^{\text{SSW}}=1.50(9)$ and $\beta_n^{\text{SDE}}=1.46(6)$ which, along with the obtained values for β_h/ν , yields $\nu^{\text{SSW}}=2.0(2)$ and $\nu^{\text{SDE}}=2.4(5)$ (note the relatively large error bar in this latter case). These values supersede the early estimate $\beta_n=1.2$ given in [13].

The scaling properties of the mean interfacial separation $\langle h \rangle$ can be determined analogously (we show results only for the SSW model). The time growth of $\langle h(t) \rangle$ for the largest system available $L=2^{17}$ yields $\theta_h^{\text{SSW}}=0.35(2)$ (see Fig. 6), in reasonable agreement with the expected value $\theta_h=1/3$. The

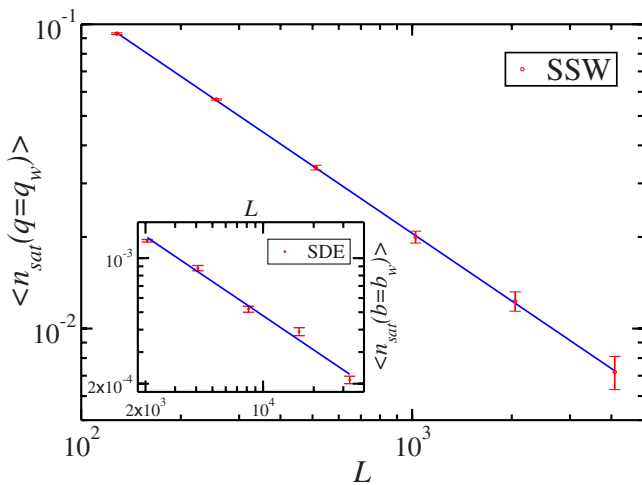


FIG. 4. (Color online) Main: The SSW saturation values of $\langle n \rangle$ at criticality for several system sizes provide the exponent ratio $\beta_h/\nu=0.74(1)$. Inset: The same analysis for the SDE yields 0.6(1). Note the difference in the magnitude of error bars, which correspond to three standard deviations of the mean saturation values.

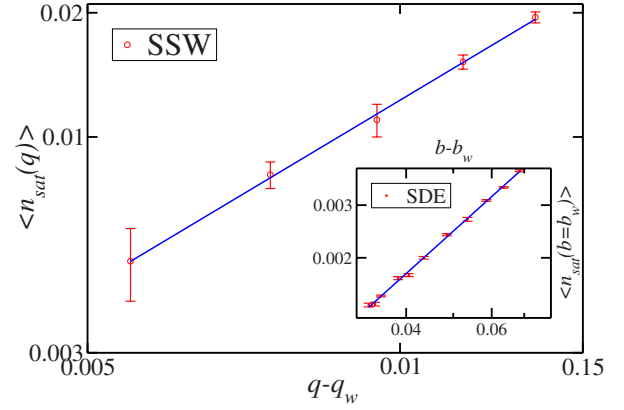


FIG. 5. (Color online) Main: The ln-ln plot of the saturation values of $\langle n \rangle$ for $L=2^{17}$ vs the distance to the critical point gives a direct estimation of $\beta_n^{\text{SSW}}=1.50(9)$ and (inset) $\beta_n^{\text{SDE}}=1.46(6)$. Error bars as in Fig. 4.

average saturation values are plotted in ln-ln as a function of the system size in the inset of Fig. 6. From them we estimate $\beta_h/\nu=0.52(4)$ (compatible with KPZ scaling, as $\zeta=\beta_h/\nu=1/2$). Estimations of z can be analogously obtained, but they are rather noisy because of the uncertainty in determining the saturation value. Also, a direct estimation of β_h can be obtained for the largest available size. This leads to $\beta_h^{\text{SSW}}=0.9(1)$ and $\beta_h^{\text{SDE}}=1.0(1)$ (see Fig. 7). Using the values of β_h/ν and β_h a third estimate for $\nu=1.8(3)$ is obtained.

Finally, we have confirmed that for both models at the tricritical point the width, W , of the interface grows with time with an exponent compatible with that of the KPZ, $W(t) \sim t^{\theta_w}$, with $\theta_w=1/3$, and saturates in finite system sizes to a nonvanishing value given by $W \sim L^{-\beta_h/\nu} \sim L^{-1/2}$ as in the KPZ.

In summary, the exponent values determined above appear to be compatible with the set of simple rational numbers $\theta_n=1/2$, $z=3/2$, $\beta_n=3/2$, $\nu=2$, $\theta_h=1/3$, and $\beta_h=1$. For the sake of comparison, the critical exponents at equilibrium

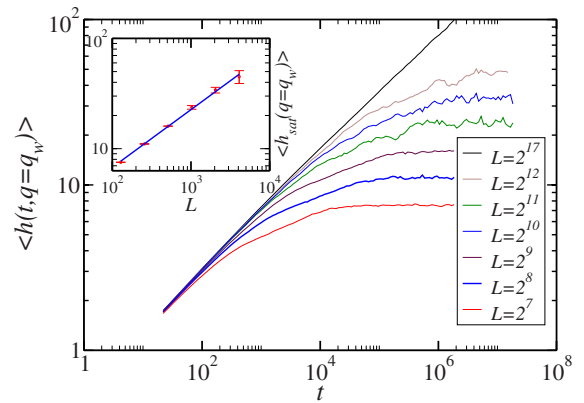


FIG. 6. (Color online) Main: The ln-ln plot of the average distance to the wall vs time, yielding $\theta_h^{\text{SSW}}=0.35(2)$. System sizes are (from top to bottom) $L=2^{17}, 2^{12}, 2^{11}, 2^{10}, 2^9, 2^8$, and 2^7 . Inset: Finite-size scaling of $\langle h_{\text{sat}}(q=q_w) \rangle$ indicating $\beta_h/\nu=0.52(4)$. All values are for the SSW model. Error bars as in Fig. 4.

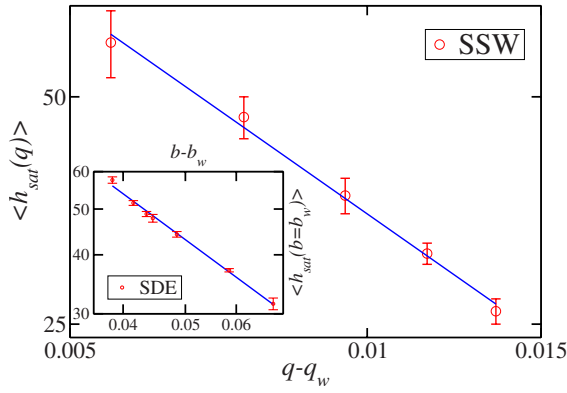


FIG. 7. (Color online) Main: The scaling of the saturation value of $\langle h_{\text{sat}} \rangle$ yields $\beta_h^{\text{SSW}}=0.9(1)$ for data collected from the SSW model. Inset: A value $\beta_h^{\text{SDE}}=1.0(1)$ results for data collected from the SDE. Error bars as in Fig. 4.

critical wetting are $\theta_n=1/4$, $z=2$, $\beta_n=1$, $\nu=2$, $\theta_h=1/4$, and $\beta_h=1$ [21]. The coincidence in the numerical values of two of them (the path dependent ones, β_h and ν) is somehow intriguing.

IV. THEORETICAL CONSIDERATIONS

It is instructive to compare the above results with those obtained at high system dimensionalities from a self-consistent, mean-field approximation to Eq. (5) [10]. In that approximation, a sequence of three scaling regimes was reported to exist depending on the relative importance of the noise strength as compared to the spatial coupling. The first two have a Gaussian character, while in the third regime, i.e., the strong noise one (in which the wetting temperature is shifted away from zero) all moments, $m_k=\langle n^k \rangle$, for $k > 2D/\sigma^2 + 1$ scale with the same exponent, while simple scaling $m_k \sim m^k$ is obtained for $k \leq 2D/\sigma^2 + 1$. This is a rather curious type of anomalous scaling not very different from that observed in analogous mean-field approximations for nonequilibrium complete wetting [22]. However, we have verified numerically that in the one-dimensional system moments of arbitrary order scale as $\langle n \rangle$ itself, as happens in other Langevin equations with multiplicative noise [9]. This is a consequence of the large fluctuations occurring at $d=1$, notwithstanding which it is possible to make some analytic predictions, as we now discuss.

First, we explicitly show how the critical value of b is depressed from its mean-field value $b_w=0$ to $b_w < 0$ once fluctuations are included. Taking spatial and noise averages in the stationary state of Eq. (4) for a generic value of a , and denoting the average squared slope of the interface by $s^2 \equiv \langle (\nabla h)^2 \rangle$, we obtain

$$a - s^2 + b\langle n \rangle + \langle n^2 \rangle = 0, \quad (6)$$

where, without loss of generality, we have taken $D=\lambda=-1$ and $c=1$. At coexistence $a_c=s_c^2$, and hence $s_c^2 - s^2 + b\langle n \rangle + \langle n^2 \rangle = 0$. Additionally, in the pinned phase (i.e., $\langle n \rangle \neq 0$) $s^2 < s_c^2$, and therefore Eq. (6) has a solution only if $b < 0$. Still,

b_w could be 0, but by noting that on approaching the tricritical point along path 5 the moments of $\langle n \rangle$ scale in the same way, $\langle n \rangle \sim \langle n^2 \rangle \sim |b - b_w|^{\beta_n}$, it is easy to see that $b_w \neq 0$ because otherwise the negative term $b\langle n \rangle$ would be subdominant in comparison with the positive $\langle n^2 \rangle$, and no solution could exist for small b . Clearly, by continuity b_w cannot be positive and hence $b_w < 0$ ensues.

It is illuminating to show how a similar reasoning leads to a more predictive analysis when applied to the analogous complete wetting transition (path 1 in Fig. 2) known to be in the MN1 class. In that case, the condition $a - a_c + s_c^2 - s^2 + b\langle n \rangle = 0$ must be satisfied, with the negative term $\delta a = a - a_c$ balancing the two positive terms $s_c^2 - s^2$ and $b\langle n \rangle \sim |\delta a|^{\beta_n}$ in the pinned phase. This implies that $\beta_n > 1$ if a solution for $\langle n \rangle$ is to exist for small δa [7]. Furthermore, by noticing that if KPZ scaling is applicable, then $s_c^2 - s^2 \sim \xi^{2(1-\zeta)}$ [23], where ζ is the roughness exponent of a free KPZ. Recalling that $\zeta=1/2$ in $1d$, immediately entails $\nu=1$ for complete wetting [7].

On the contrary, we have not been able to derive the value of ν for nonequilibrium critical wetting, nor does it seem immediate that $\nu=1$ along path 4 as found numerically in [12]. The main difference with the complete wetting case appears to rest on the behavior of the slopes s^2 . According to our measurements for Eq. (4), $s_c^2 - s^2 \sim |b - b_w|^{1.51(2)}$ for critical wetting (path 5) and $s_c^2 - s^2 \sim |a - a_c|^{0.74(2)}$ for tricritical complete wetting (path 4), pointing to an anomalous scaling of the slopes, i.e., $s^2 = \langle (\nabla h)^2 \rangle$ does not have the same scaling dimension as $[L^{-1}]^2[h]^2$, as happens in the complete wetting case.

That $s_c^2 - s^2$ scales along path 4 with an exponent less than unity could have been anticipated from the condition $|\delta a| - b_w |\delta a|^{\beta_n} = s_c^2 - s^2$ at the tricritical point, implying $s_c^2 - s^2 \sim |\delta a|^{\beta_s}$ with $\beta_s \leq 1$.

The exponent value $\beta_n=0.74(5)$ reported in [12] for the multicritical complete wetting transition along path 4 (confirmed in our own measurements), along with $\beta_n \approx 1.5$ for critical wetting as referred to above, indicates that the anomalous scaling of the slopes is ultimately controlled by β_n in both cases, rather than by the roughness exponent of the KPZ as in the complete wetting case. Notice the constancy along paths 4 and 5 of the ratio $\beta_n/\nu \approx 0.75$, which is a property of the tricritical point and therefore must be independent of the path.

The fact that the slopes acquire a scaling not directly derivable from the free KPZ can be argued to be a consequence of the effect of the potential well on the interface. Figure 8 shows snapshots of configurations $h(x)$ that result from solving the SDE in the stationary state for different parameters that correspond to approaching the critical points from different paths:

Panel (a) for $(b=-1.2 < b_w, a=-0.25 < a_c)$, corresponds with a situation slightly below the complete wetting (MN1) transition (path 1 of Fig. 2).

Panel (b) for $(b=b_w, a=-0.153 < a_c)$, slightly below the multicritical complete wetting transition (path 4).

Panel (c) for $(b=-1.3 < b_w, a=a_c)$, slightly below the critical wetting transition (path 5).

The distances to the transition points are chosen so that $\langle h \rangle \approx 5$ in all cases. Note the clear qualitative difference be-

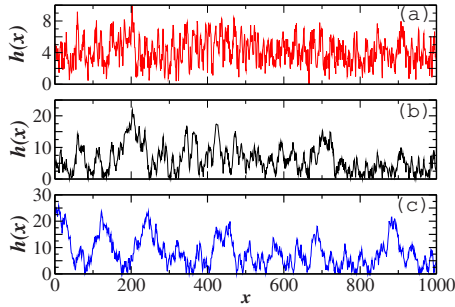


FIG. 8. (Color online) Snapshots of configurations $h(x)$ as a result from solving the SDE in the stationary state for (a) $b = -1.2 < b_w$, $a = -0.25 < a_c$ (complete wetting), (b) $b = b_w$, $a = -0.153 < a_c$ (multicritical complete wetting), and (c) $b = -1.3$, $a = a_c$ (critical wetting). The distances to the transition points are chosen so that $\langle h \rangle \approx 5$ in all cases. In panels (b) and (c) patches of depinned interfaces are observed where $n \approx 0$.

tween panel (a), for which standard scaling holds, and the rest. Observe that in panel (c), the interface consists of patches of essentially free KPZ interfaces separated by regions of sites pinned by the potential well. It then seems plausible to conclude that, following path 5, regions locally trapped within the potential well develop slopes different from that of a free KPZ (indeed, roughness is severely restricted within the potential well) even at points arbitrarily close to the tricritical point, thereby inducing an anomalous scaling controlled by β_n . In panel (b), i.e., upon approaching the tricritical point along path 4, the effect of the bounding wall is less apparent as the potential well is marginally disappearing at $b = b_w$, but a similar effect, induced by the potential shape, should be at work in this borderline case.

V. SUMMARY AND DISCUSSION

We have investigated the universal properties of nonequilibrium critical wetting transitions in one spatial dimension. For that, we study effective interfacial models in the KPZ universality class with $\lambda < 0$ bounded by a lower wall, and determine the average height $\langle h \rangle$ and the surface order parameter $\langle n \rangle = \langle e^{-h} \rangle$.

A scaling analysis leads to the prediction $z = 3/2$ and a time behavior of the average height governed by the growth exponent of the free KPZ, i.e., $\langle h(t) \rangle \sim t^{1/3}$. These results have been verified numerically. Other exponents have been computed from extensive numerical simulations of the Langevin equation and a discrete model in the same universality class that enables a more precise numerical analysis, as a result of which we find $\nu \approx 2$, $\beta_h \approx 1$, and $\beta_n \approx 3/2$, suggesting that actually the exponents take rational values (see Table I). Interestingly enough, the first two values agree with those of equilibrium critical wetting.

Simple analytical arguments allow us to show that the critical value of the control parameter b is depressed by fluctuations from its mean-field value $b_w = 0$ to $b_w < 0$. We have also shown that in critical wetting, as well as in multicritical complete wetting, the average interface slopes do show

TABLE I. Summary of the critical exponents for nonequilibrium, critical wetting transitions with short-range forces. Results are shown for the discrete model SSW, the stochastic differential equation (SDE), and from other sources when available.

Exponent	SSW	SDE	Others
z	1.4(1)	1.3(2)	1.5(1) [11]
θ_n	0.49(2)	0.50(5)	0.50(1) [11]
θ_h	0.35(2)	0.33(1)	
ν	2.0(2)	2.4(5)	
β_n	1.50(9)	1.46(6)	
β_h	0.9(1)	1.0(1)	

anomalous scaling, not controlled by the free KPZ equation: local regions pinned by the binding potential generate anomalous scaling.

We have not been able to predict the value of ν using the same analytical considerations that yield $\nu = 1$ in the complete wetting (MN1) case, nor is it trivial to obtain $\nu = 1$ for multicritical complete wetting. It is nevertheless possible to conclude that in this latter case the exponent governing the scaling of the interface slopes is less than unity, $s_c^2 - s^2 \sim |a - a_c|^{\beta_s}$, in agreement with the value $\beta_s \approx 0.75$ obtained. There is numeric evidence that the slopes actually scale with $\beta_s = \beta_n$ for both critical wetting and multicritical complete wetting (paths 5 and 4 of Fig. 2, respectively), a result that violates naive scaling. In effect, dimensional analysis demands that $s_c^2 - s^2$ scales as $|a - a_c|^{2(1-\zeta)}$, ζ being the roughness exponent of the free KPZ [23]. This is obeyed at the complete wetting transition, and indeed was used to derive $\nu = 1$ in MN1 [7], but does not hold for critical wetting nor for multicritical complete wetting.

The cause of the deviation from standard scaling can be sought in the interfacial profiles shown in Fig. 8. For critical and multicritical wetting, patches of free KPZ interfaces [$n(x) \approx 0$] are separated by regions of sites that lie in the potential well, plausibly hindering the standard KPZ scaling from setting in gradually as the tricritical point is approached. This is in contrast to typical interfacial profiles for nonequilibrium complete wetting (MN1) where no potential well is at play.

According to a recent self-consistent, mean-field approximation to Eq. (5) three different scaling regimes of critical behavior for the surface order parameter can be distinguished [10]. The first two are of Gaussian type, while the third one is a highly nontrivial strong-fluctuating regime. This rich structure is completely washed out by fluctuations in one-dimensional systems, where a unique and universal scaling regime emerges. Moreover, we have verified numerically that in the one-dimensional system moments of arbitrary order scale as $\langle n \rangle$ itself, as happens in other Langevin equations with multiplicative noise [9]. The fact that the numerical values of the exponents are changed is not surprising at all, given the presence of severe fluctuations in one dimension but what is more striking is that out of the three regimes appearing in the mean-field approach, only one survives.

A possible explanation for such an abrupt change might come from a recent claim that the strong-coupling renormalization group fixed point of the KPZ dynamics is essentially different above and below $d=2$ [24]. This point remains to be further studied, as well as some other aspects of nonequilibrium critical wetting including its subtle relation to its equilibrium counterpart (signaled by the coincidence of some exponents), and the possible existence of various nonuniversal scaling regimes in higher dimensions. It also stands as a main experimental challenge to observe in the laboratory the

phenomenology reported on here and in previous theoretical works of nonequilibrium wetting.

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